Dynamic Mechanisms with Budget Balance

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1 Bilateral Trade Model

A seller (S) with a hidden cost c wants to a good to a buyer (B) with hidden value v. Priors: $v \sim F$ on $\mathcal{V} = [\underline{v}, \overline{v}]$ and $c \sim G$ on $\mathcal{C} = [\underline{c}, \overline{c}]$. For allocation rule (probability of trade) p, and transfers x_B and x_S, payoffs are given by

$$\textbf{Buyer:} \quad vp - x_B, \qquad \textbf{Seller:} \quad x_S - cp$$

Three standard constraints imposed on this mechanism design problem are: incentive compatibility, individual rationality and budget balance. While the first is an equilibrium condition, the other two are institutional constraints. The standard objective tend to be efficiency or maximization of gains from trade.

The efficient allocation is defined as

$$p^*(v,c) = \begin{cases} 1 & \text{if } v > c \\ 0 & \text{otherwise} \end{cases}$$

A famous result by Myerson and Satterthwaite [1983] shows that there does not exist any mechanism that satisfies all the three constraints while implementing the efficient allocation.

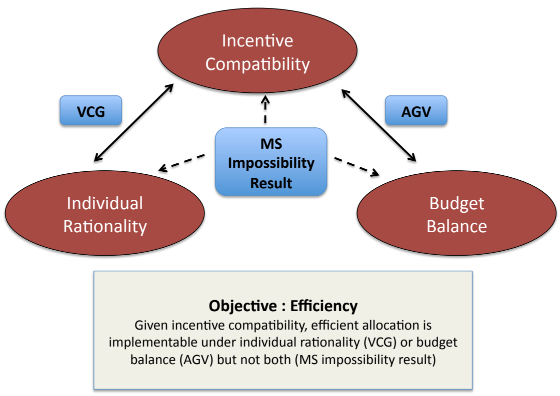

Figure 1: Interaction of constraints

The problem of maximization of gains from trade can be written as

$$\max_{p,x} \quad \int_{\underline{v}}^{\overline{v}} \int_{\overline{c}}^{\underline{c}} (v-c)\, p(v,c)g(c)dcf(v)dv$$

subject to

$$IC_B(v), IR_B(v), IC_S(c), IR_S(c), BB$$

A relaxed problem (ignoring) monotonicity constraints can be re-written as

$$(RP) \quad \max_{p,x} \quad \int_{\underline{v}}^{\overline{v}} \int_{\overline{c}}^{\underline{c}} (v-c)\, p(v,c)g(c)dcf(v)dv$$

subject to

$$\int_{\underline{v}}^{\overline{v}} \int_{\overline{c}}^{\underline{c}} \left[\left(v - \frac{1-F(v)}{f(v)} \right) - \left(c + \frac{G(c)}{g(c)} \right) \right] p(v,c)g(c)dcf(v)dv \geq 0$$

that is,

$$(RP) \quad \max_{p,x} \quad S$$

subject to

$$VS \geq 0$$

2 Uniform Trading Model

Assume types of both agents are uniformly distributed on $[0,1]$. We want to maximize gains from trade. That is the optimization problem is given by

$$\max_{p,x} \quad \int_0^1 \int_0^1 (v-c)\, p(v,c)dcdv$$

subject to

$$\int_0^1 \int_0^1 \left[(2v-1) - 2c \right] p(v,c)dcdv \geq 0$$

The solution is give by: trade if and only if $v \geq c + M$, where M solves

$$\frac{1}{6}(4M-1)(1-M)^2 = 0$$

3 Two-period Uniform Trading Problem

Consider the same problem with IID types distributed uniformly on the unit interval first under ex ante budget balance and then under interim budget balance.

Under ex ante budget balance the problem can be stated as

$$\max_{p,x} \quad (1+\delta) \int_0^1 \int_0^1 (v-c)\, p(v,c)dcdv$$

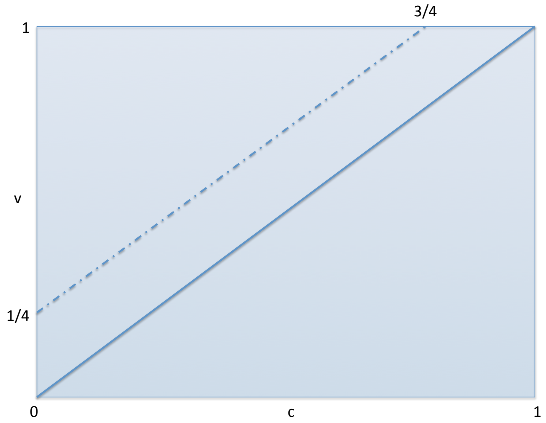

Figure 2: Static Bilateral Trade Problem

subject to

$$\int_0^1 \int_0^1 \left[(2v-1)-2c\right] p(v,c)dcdv + \delta \int_0^1 \int_0^1 (v-c)p(v,c)dcdv \geq 0$$

In the second period trade is always efficient. In the first period trade is given by the linear trading rule: $v_1 \geq c_1 + M$, where M solves

$$\frac{1}{6}(4M-1)(1-M)^2 + \delta\frac{1}{6} = 0$$

Under interim budget balance the problem can be stated as

$$\max_{p,x} \quad (1+\delta) \int_0^1 \int_0^1 (v-c)\, p(v,c)dcdv$$

subject to

$$\int_0^1 \int_0^1 \left[(2v-1)-2c\right] p(v,c)dcdv + \delta \int_0^1 \int_0^1 (v-c)p(v,c)dcdv \geq 0$$

In the second period trade replicates the static bilateral trade problem: $v_2 \geq c_2 + \frac{1}{4}$. In the first period trade is given by the linear trading rule: $v_1 \geq c_1 + M$, where M solves

$$\frac{1}{6}(4M-1)(1-M)^2 + \delta\frac{9}{64} = 0$$

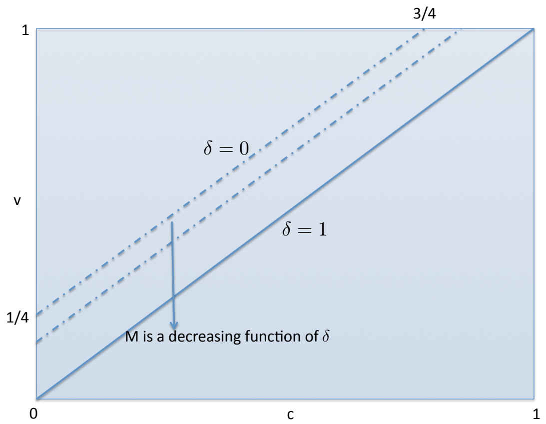

Figure 3: Repeated Bilateral Trade Problem with Ex ante Budget Balance

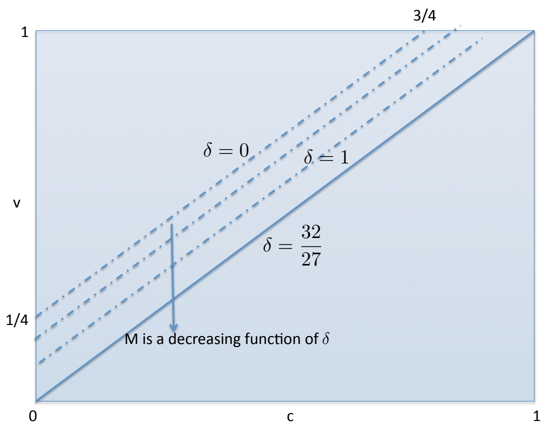

Figure 4: Repeated Bilateral Trade Problem with Interim Budget Balance